# AP® Calculus 2008 Summer Assignment

Complete the following problems, showing full supporting analyses. Problems designed for the graphing calculator have been designated as such and do not require additional written work.

For problems 1 – 4, find all intercepts (if any).

1. $y = 2x - 3$
2. $y = (x-1)(x-3)$
3. $y = \dfrac{x-1}{x-2}$
4. $xy = 4$

For problems 5 and 6, check for symmetry with respect to both axes and to the origin.

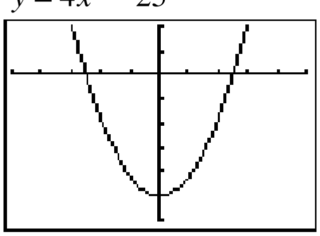
5. $x^2 y - x^2 + 4y = 0$
6. $y = x^4 - x^2 + 3$

For problems 7 – 14, sketch the graph of the equation by hand. You may use a graphing calculator only to check that your answer is correct!

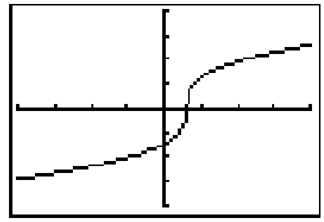
7. $y = \frac{1}{2}(-x+3)$
8. $4x - 2y = 6$
9. $-\frac{1}{3}x + \frac{5}{6}y = 1$
10. $0.02x + 0.15y = 0.25$
11. $y = 7 - 6x - x^2$
12. $y = 6x - x^2$
13. $y = \sqrt{5-x}$
14. $y = |x-4| - 4$

For problems 15 and 16, describe the exact viewing window of a graphing calculator that produces the figure as it's shown.

15. $y = 4x^2 - 25$



16. $y = 8\sqrt[3]{x-6}$



For problems 17 and 18, use a graphing calculator to determine the point(s) of intersection of the graphs of the equations.

17. $\begin{cases} 3x - 4y = 8 \\ x + y = 5 \end{cases}$

18. $\begin{cases} x - y + 1 = 0 \\ y - x^2 = 7 \end{cases}$

For problems 19 and 20, *think about it*. Use a numerical, analytical, or graphical approach to explore the situation and devise your solution.

19. Write an equation whose graph has intercepts at $x = -2$ and $x = 2$ and is symmetric with respect to the origin.

20. For what value of $k$ does the graph of $y = kx^3$ pass through the point, for each point given below?
    a. $(1,4)$
    b. $(-2,1)$
    c. $(0,0)$
    d. $(-1,-1)$

For problems 21 and 22, plot the points and find the slope of the line passing through the points

21. $\left(\frac{3}{2},1\right)$ and $\left(5,\frac{5}{2}\right)$

22. $(7,-1)$ and $(7,12)$

For problems 23 and 24, use the concept of slope to find $t$ such that the three points are collinear.

23. $(-2,5),(0,t),(1,1)$

24. $(-3,3),(t,-1),(8,6)$

For problems 25 – 28, find an equation of the line that passes through the point with the indicated slope. Sketch the line.

25. $(0,-5)$, $m=\frac{3}{2}$

26. $(-2,6)$, $m=0$

27. $(-3,0)$, $m=-\frac{2}{3}$

28. $(5,4)$, $m$ is undefined.

For problems 29 and 30, write an equation for each part of the problem.

29. Find equations of the lines passing through $(-2,4)$ and having the following characteristics.
   a. Slope of $\frac{7}{16}$
   b. Parallel to the line $5x-3y=3$
   c. Passing through the origin
   d. Parallel to the $y$-axis.

30. Find equations of the lines passing through $(1,3)$ and having the following characteristics.
   a. Slope of $-\frac{2}{3}$
   b. Perpendicular to the line $x+y=0$
   c. Passing through the point $(2,4)$
   d. Parallel to the $x$-axis

For problems 31 and 32, apply linear functions to model real-world situations.

31. The purchase price of a new machine is $12,500 and its value will decrease by $850 per year. Write a linear equation that gives the value $V$ of the machine $t$ years after it is purchased. Find its value at the end of 3 years.

32. A contractor purchases a piece of equipment for $36,500 that costs an average of $9.25 per hour for fuel and maintenance. The equipment operator is paid $13.50 per hour, and customers are charged $30 per hour.
   a. Write an equation for the cost $C$ of operating this equipment for $t$ hours.
   b. Write an equation for the revenue $R$ derived from $t$ hours of use.
   c. Find the break-even point for this equipment by finding the time at which $R=C$.

For problems 33 – 36, sketch the graph of the equation and use the Vertical Line Test to determine whether the equation expresses $y$ as a function of $x$.

33. $x-y^2=0$

34. $x^2-y=0$

35. $y=x^2-2x$

36. $x=9-y^2$

For problems 37 – 41, evaluate, analyze, and/or graph functions using function notation.

37. Evaluate (if possible) the function $f(x)=\frac{1}{x}$ at the specified values of the independent variable, and simplify the results.
   a. $f(0)$
   b. $\frac{f(1+h)-f(1)}{h}$

38. Evaluate (if possible) the function at each value of the independent variable.
$$f(x)=\begin{cases} x^2+2, & x<0 \\ |x-2|, & x\geq 0 \end{cases}$$
   a. $f(-4)$
   b. $f(0)$
   c. $f(1)$

39. Find the domain and range of each function below.

a. $y=\sqrt{36-x^2}$

b. $y=\dfrac{7}{2x-10}$

c. $y=\begin{cases} x^2, & x<0 \\ 2-x, & x\geq 0 \end{cases}$

40. Given $f(x)=1-x^2$ and $g(x)=2x+1$, evaluate each expression below.

a. $f(x)-g(x)$

b. $f(x)g(x)$

c. $g(f(x))$

d. $f(g(x))$

41. Sketch three graphs on the same set of coordinate axes graphs of $f$ for the values $c=-2,0,2$, for each function below.

a. $f(x)=x^3+c$

b. $f(x)=(x-c)^3$

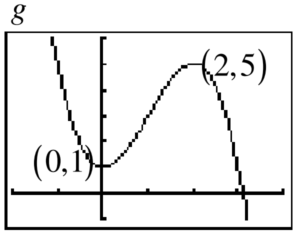
c. $f(x)=(x-2)^3+c$

d. $f(x)=cx^3$

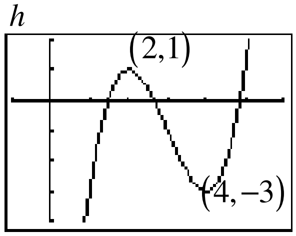
For problems 42 and 43, use a graphing calculator to explore each problem.

42. Use a graphing calculator to graph $f(x)=x^3-3x^2$. Use the graph to write a formula for the functions $g$ and $h$ shown below.

a. $g$




b. $h$




43. A wire 24 inches long is to be cut into four pieces to form a rectangle whose shortest side has length $x$.

a. Write the area $A$ of the rectangle as a function of $x$.

b. Determine the domain of the function and use a graphing calculator to graph the function over that domain.

c. Use the graph of the function to approximate the maximum area of the rectangle. Make a conjecture about the dimensions that yield a maximum area.

For problems 44 and 45, use properties of circles and lines to solve each problem.

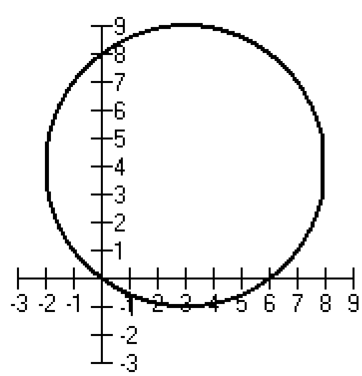
44. Consider the circle $x^2+y^2-6x-8y=0$ as shown below.

a. Determine its center and radius

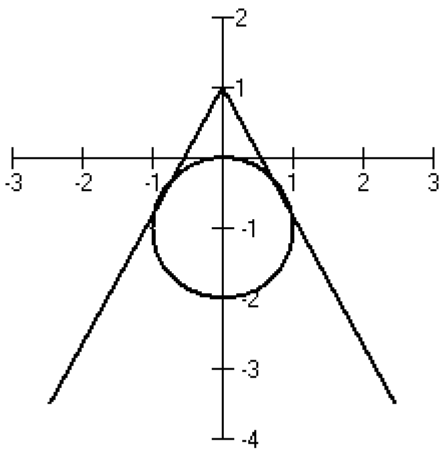
b. Find an equation for the tangent line to the circle at the point $(0,0)$.

c. Find an equation for the tangent line to the circle at the point $(6,0)$.

d. Where do the two tangent lines intersect?



45. There are two tangent lines from the point $(0,1)$ to the circle $x^2+(y+1)^2=1$ as shown in the figure below. Find equations of these two lines by using the fact that each tangent line intersects the circle in exactly one point.




For problems 46 and 47, use function transformations to sketch each graph.

46. The Heaviside function $H(x)$ is widely used in engineering applications and is defined by the formula $H(x)=\begin{cases}1, & x\ge 0\\ 0, & x<0\end{cases}$. Sketch the graph of the Heaviside function and the graphs of the following functions.

a. $H(x)-2$

b. $H(x-2)$

c. $-H(x)$

d. $H(-x)$

e. $\frac{1}{2}H(x)$

f. $-H(x-2)+2$

47. Consider the graph of the function $f$ shown below. Use this graph to sketch the graphs of the following functions.

a. $f(x+1)$

b. $f(x)+1$

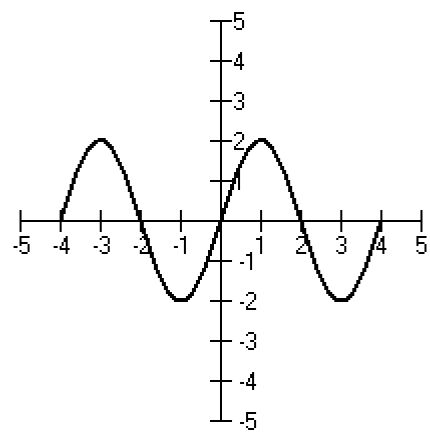
c. $2f(x)$

d. $f(-x)$

e. $-f(x)$

f. $|f(x)|$

g. $f(|x|)$




For problems 48 – 50, represent each application with an algebraic function.

48. A rancher plans to fence a rectangular pasture adjacent to a river. The rancher has 100 meters of fence, and no fencing is needed along the river.

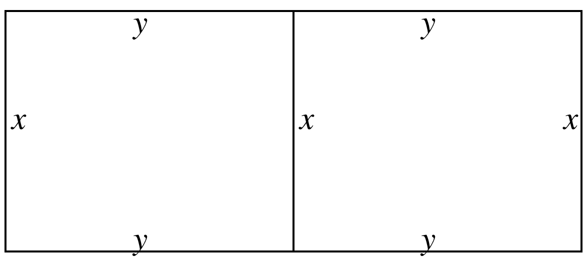
a. Write the area $A$ of the pasture as a function of $x$, the length of the side parallel to the river. What is the domain of $A$?

b. Graph the area function $A(x)$ and estimate the dimensions that yield the maximum amount of area for the pasture on your calculator.

c. Find the dimensions that yield the maximum amount of area for the pasture algebraically.

49. A rancher has 300 feet of fence to enclose two adjacent pastures.

   a. Write the total area $A$ of the two pastures as a function of $x$ (see figure). What is the domain of $A$?
   b. Graph the area function and estimate the dimensions that yield the maximum amount for the pastures on your calculator.
   c. Find the dimensions that yield the maximum amount of area for the pastures algebraically.




50. You are in a boat 2 miles from the nearest point on the coast. You are to go to a point Q located 3 miles down the coast and 1 mile inland. You can row at 2 miles per hour and walk at 4 miles per hour. Write the total time $T$ of the trip as a function of $x$ (see figure).