## **Honors Algebra**: **Field Properties Enrichment** Name ______________________________

1. Suppose a special form of addition, denoted by $\oplus$, is defined for the set $\{0, 1, 2, 3\}$ as follows: If $a$ and b are any two elements of the set, then the sum $a \oplus b$ is the remainder when the "usual" sum of $a + b$ is divided by 4. For example, $2 \oplus 3 = 1$, because 1 is the remainder when $2 + 3$, or 5, is divided by 4. Use the definition of $\oplus$ to complete the addition table at the right.

| $\oplus$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | 1 |
| 3 | | | | |

2. Suppose a special form of multiplication, denoted by $\otimes$, is defined for the set $\{0, 1, 2, 3\}$ as follows: If $a$ and $b$ are any two elements of the set, then the product $a \otimes b$ is the remainder when the "usual" product $ab$ is divided by 4. For example, $2 \otimes 3 = 2$, because 2 is the remainder when $2 \times 3$, or 6, is divided by 4. Use the definition of $\otimes$ to complete the multiplication table at the right.

| $\otimes$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | 2 |
| 3 | | | | |

3. Based on the results of the two problems above, is the set $\{0, 1, 2, 3\}$ ***closed*** for $\oplus$ and $\otimes$? Explain.

4. Based on the results of problem 1, what number from the set $\{0, 1, 2, 3\}$ is the ***opposite*** of 3 under $\oplus$? Explain.

5. Based on the results of problem 2, what number from the set $\{0, 1, 2, 3\}$ is the ***reciprocal*** of 3 under $\otimes$? Explain.

6. Determine whether or not the set $\{0,1,2,3\}$ and the operations $\oplus$ and $\otimes$ constitute a ***field***. Defend your answer.

7. For the set $\{0,1,2,3\}$, create a definition for a special form of subtraction, denoted by $\Theta$.

8. Use your definition to complete the subtraction table at the right.

| $\Theta$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |