FORMULAS → SOLUTIONS ← INSTANTANEOUS / PICTURES

1. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(4)^2\left(\frac{-1}{2}\right)$

$\frac{dV}{dt} = -32\pi \ cm^3/min$

$\frac{dr}{dt} = -\frac{1}{2} \frac{cm}{min}$

@ $r=4$

2. $A = \pi r^2$

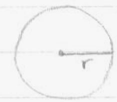
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$3 = 2\pi(10)\left(\frac{dr}{dt}\right)$

$\frac{3}{20\pi} = \frac{dr}{dt}$

$\frac{dA}{dt} = 3 \frac{cm^2}{min}$

@ $r = 10$



3. $x^2 + y^2 = 9$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

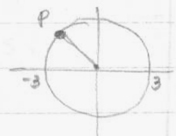
$2(-\sqrt{3})(20) + 2(\sqrt{6})\frac{dy}{dt} = 0$

$2\sqrt{6}\ \frac{dy}{dt} = 40\sqrt{3}$

$\frac{dy}{dt} = \frac{40\sqrt{3}}{2\sqrt{6}} = 10\sqrt{2}\ \frac{ur}{sec}$

$P = (-\sqrt{3}, \sqrt{6})$

$\frac{dx}{dt} = 20$



4. $x^2 + h^2 = 225$

$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 0$

$2(5)\left(-\frac{1}{2}5\right) + 2(10\sqrt{2})\frac{dh}{dt} = 0$

$20\sqrt{2}\ \frac{dh}{dt} = 25$

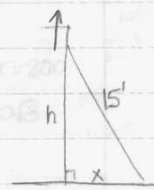
$\frac{dh}{dt} = \frac{25}{20\sqrt{2}} = \frac{5\sqrt{2}}{8}\ \frac{ft}{sec.}$

@ $x=5$

$25 + h^2 = 225$

$h^2 = 200$

$h = 10\sqrt{2}$



$\leftarrow -\frac{1}{2}x \Rightarrow \frac{dx}{dt}$

5. $V = \left(\frac{1}{2}bh\right)l$

$V = \frac{1}{2}\left(\frac{2}{\sqrt{3}}h\right)h(12)$

$V = 4\sqrt{3}h^2$

$\frac{dV}{dt} = 8\sqrt{3}h \cdot \frac{dh}{dt}$

$3 = 8\sqrt{3}\left(\frac{1}{2}\right)\left(\frac{dh}{dt}\right)$

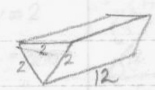
$3 = 4\sqrt{3}\ \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3}{4\sqrt{3}} = \frac{\sqrt{3}}{4}\ \frac{ft}{min.}$

$\frac{dV}{dt} = 3 \frac{ft^3}{min}$

@ $h = .5ft$

$\frac{dh}{dt}$?



$\frac{b}{h} = \frac{2}{\sqrt{3}}$

$b = \frac{2h}{\sqrt{3}}$

6. $\tan\theta = x$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$

$(2)^2 \cdot (10\pi) = \frac{dx}{dt}$

$\frac{dx}{dt} = 40\pi \ mi/min$

$\frac{d\theta}{dt} = 10\pi$

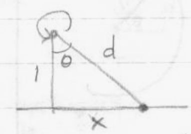
@ $d=2$

$x^2 + 1 = 4$

$x^2 = 3 \quad x = \sqrt{3}$

$\cos\theta = \frac{1}{2}$

$+ \sec\theta = 2$



7. $y = 5001 - 2500\sqrt{4-x}$

$\frac{dy}{dt} = -2500 \cdot \frac{1}{2}(4-x)^{-1/2}\left(-\frac{dx}{dt}\right)$

$\frac{dy}{dt} = \frac{1250}{\sqrt{4-x}} \cdot \frac{dx}{dt}$

a) $\frac{dy}{dt} = \frac{1250}{\sqrt{4-3}}(10) = 12{,}500\ \frac{ft}{min}$

b) When tortoise crosses the line $\Rightarrow x=0$.

When $x=0$, $y = 5001 - 2500\sqrt{4-0} =$

$= 5001 - 5000$

$= 1 \Rightarrow$ still has 1 ft. to go to finish line

∴ the Tortoise wins by 1 foot.

T ⟶ $dx/dt = 1$

H ⟶ $\frac{dy}{dt} = ?$

8. $p^2+s^2=d^2$

$(\frac{1}{4})^2+s^2=d^2$

$0+2s\frac{ds}{dt}=2d\frac{dd}{dt}$

$2(\frac{1}{8})(-40)=2(\frac{\sqrt{5}}{8})\frac{dd}{dt}$

$-10=\frac{\sqrt{5}}{4}\cdot\frac{dd}{dt}$

$\frac{dd}{dt}=-8\sqrt{5}$ mph decreasing!

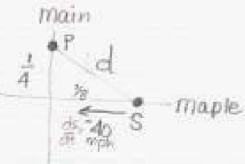
@ $s=\frac{1}{8}$

$(\frac{1}{4})^2+(\frac{1}{8})^2=d^2$

$\frac{1}{16}+\frac{1}{64}=d^2$

$\frac{5}{64}=d^2$

$d=\frac{\sqrt{5}}{8}$



9. $p^2+s^2=d^2$

$(\frac{1}{4})^2+s^2=d^2$

$2s\frac{ds}{dt}=2d\frac{dd}{dt}$

$2s(-50)=2d(-30)$

$100s=60d$

$\frac{5}{3}s=d$

$(\frac{1}{4})^2+s^2=(\frac{5}{3}s)^2$

$\frac{1}{16}+s^2=\frac{25}{9}s^2$

$\frac{1}{16}=\frac{16}{9}s^2$

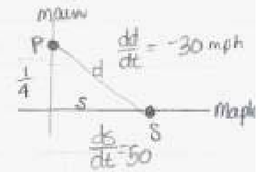
$\frac{9}{16^2}=s^2$

$\therefore s=\frac{3}{16}$ mi

$\frac{dd}{dt}=-30$

$\frac{ds}{dt}=-50$

find s!



10. $100^2+x^2=r^2$

$2x\frac{dx}{dt}=2r\frac{dr}{dt}$

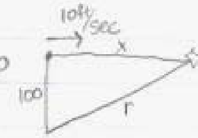
$2(100\sqrt{3})(10)=2(200)\frac{dr}{dt}$

$2000\sqrt{3}=400\frac{dr}{dt}$

$\frac{dr}{dt}=5\sqrt{3}$ ft/sec

@ $r=200$

$x=100\sqrt{3}$



11. $\tan\theta=\frac{y}{5}$

$\sec^2\theta\cdot\frac{d\theta}{dt}=\frac{1}{5}\frac{dy}{dt}$

$\sec^2(.3805)\cdot\frac{d\theta}{dt}=\frac{1}{5}(300)$

$1.16\frac{d\theta}{dt}=60$

$\frac{d\theta}{dt}=51.72$ or $\frac{1500}{29}$ rad/hr.

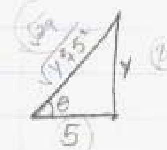
@ $y=2$

$\frac{dy}{dt}=300$

$\tan\theta=\frac{2}{5}$

$\theta=.3805$ rad. or

$\sec\theta=\frac{\sqrt{29}}{5}\rightarrow\sec\theta\approx1.0770$



12. $\sin\theta=\frac{y}{125}$

$\cos\theta\cdot\frac{d\theta}{dt}=\frac{1}{125}\frac{dy}{dt}$

$125\cos\theta\cdot\frac{d\theta}{dt}=\frac{dy}{dt}$

$125\cdot\frac{4}{5}\cdot2=\frac{dy}{dt}$

$200=\frac{dy}{dt}$

$\therefore$ It's rising at 200 ft/min.

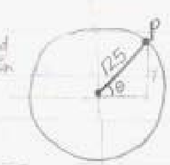
$\frac{d\theta}{dt}=2\frac{\text{rad}}{\text{min}}$

@ $y=75$

$\frac{dy}{dt}$

$\rightarrow\sin\theta=\frac{75}{125}=\frac{3}{5}$

$\rightarrow\cos\theta=\frac{4}{5}$ (3,4,5)



13. $x^2+5^2=r^2$

$2x\frac{dx}{dt}=2r\frac{dr}{dt}$

$2(12)\frac{dx}{dt}=2(13)(2)$

$24\frac{dx}{dt}=52$

$\frac{dx}{dt}=\frac{13}{6}$ ft/sec.

@ $r=13$

$x=12$

14. $V = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2(6.67\times10^{-11})(4\times10^{30})}{r}} = \sqrt{\frac{5.336\times10^{20}}{r}}$ $\quad M = 4\times10^{30}$, $G = 6.67\times10^{-11}$

$$= \frac{2.31\times10^{10}}{\sqrt{r}} = (2.31\times10^{10})\,r^{-1/2}$$

@ $r = 45{,}000$, $\frac{dr}{dt} = -3\times10^{-6}$

$V = (2GM\cdot r^{-1})^{1/2}$

$$\frac{dV}{dt} = \frac{1}{2}(2GM\cdot r^{-1})^{-1/2}\left(-2GM\,r^{-2}\frac{dr}{dt}\right)$$

$$= \frac{1}{2\sqrt{2GM/r}}\left(\frac{-2GM}{r^2}\cdot\frac{dr}{dt}\right)$$

$\sqrt{r}$ $\quad \sqrt{GM}\sqrt{GM}$ $\quad r\cdot\sqrt{r}\sqrt{r}$

$$= \frac{-\sqrt{GM}}{r\sqrt{2r}}\cdot\frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{-\sqrt{(6.67\times10^{-11})(4\times10^{30})}}{(4.5\times10^{3})\sqrt{9\times10^{4}}}\left(-3\times10^{-6}\right)$$

$$= 3.63\times10^{-2}$$

$\frac{dV}{dt} = .0363$ km/sec.^{2}

CHECK!

$$\frac{\sqrt{r}}{\sqrt{2GM}}\cdot\frac{-GM}{r^2}\cdot\frac{dr}{dt}$$

$$\frac{\cancel{\sqrt{r}}}{\sqrt{2}\cdot\cancel{\sqrt{GM}}}\cdot\frac{-\sqrt{GM}\cdot\cancel{\sqrt{GM}}}{r\cdot\cancel{\sqrt{r}}\cdot\sqrt{r}}\cdot\frac{dr}{dt}$$

$$\frac{-\sqrt{GM}}{r\cdot\sqrt{2r}}\cdot\frac{dr}{dt}$$